Exercise 1.1

1. Provide a definition of an image, along with different mathematical representations.
2. List five different applications of image processing.
3. Provide a definition of image processing and its different levels.
4. Explain each of the following concepts in image processing: Quantization; Spatial resolution; Digitization; Tonal resolution; Pixel; Sampling.
5. The digitization of an image is divided into two steps. Explain each of these steps and their effects on the appearance of the digitized image.
6. Why are image dimensions often 512×512, 256×256, 128×128?
7. Provide a definition of noise in an image.
8. How many gray levels are possible in a color image encoded with 12 bits?
9. How many bytes are needed to store, without compression (header is neglected): a color image of size 512×256, encoded in 16 bits ; a 3D image of size $256 \times 256 \times 128$ in grayscale, where each pixel is encoded in 32 bits?
10. Explain the principle of steganography.
11. What is image registration?

Exercise 1.2

An image of size 4×4, with 4 bits per pixel, has undergone the following degradation:

$$I = \begin{array}{|c|c|c|c|}\hline 8 & 3 & 7 & 3 \\ \hline 4 & 11 & 15 & 12 \\ \hline 0 & 10 & 11 & 1 \\ \hline 2 & 10 & 3 & 6 \\ \hline \end{array} \quad \rightarrow \quad J = \begin{array}{|c|c|c|c|}\hline 2 & 1 & 7 & 3 \\ \hline 1 & 1 & 15 & 12 \\ \hline 0 & 13 & 5 & 13 \\ \hline 2 & 10 & 4 & 6 \\ \hline \end{array} .$$

Calculate the errors (MAE), (MSE), and (PSNR)[1] between the original image (I) and the degraded image (J).

Exercise 1.3

(1) Determine the memory (in bytes) necessary for storing the following images:

[1]The PSNR (in dB) is defined by: $10 \log_{10}\left(\frac{MAX_I^2}{MSE}\right)$

– Binary image 64×64;

– 8-bit grayscale image 128×128;

– 24-bit color image 64×64;

– Binary image 512×512;

– 8-bit grayscale image 1024×1024;

– 24-bit color image 4096×4096.

(2) Answer the previous questions after applying the following operations:

– Double the number of rows and columns;

– Reduce the number of rows and columns by 2.

Exercise 1.4

Find the bit-planes of each of the following images and verify the reconstruction.

$$I_1 = \begin{array}{|c|c|c|c|}\hline 8 & 3 & 7 & 3 \\ \hline 4 & 11 & 15 & 12 \\ \hline 0 & 10 & 11 & 1 \\ \hline 2 & 10 & 3 & 6 \\ \hline \end{array} \quad , \quad I_2 = \begin{array}{|c|c|c|c|}\hline 2 & 1 & 7 & 3 \\ \hline 1 & 1 & 15 & 12 \\ \hline 0 & 13 & 5 & 13 \\ \hline 2 & 10 & 4 & 6 \\ \hline \end{array} \quad , \quad I_3 = \begin{array}{|c|c|c|c|}\hline 8 & 1 & 7 & 8 \\ \hline 5 & 11 & 15 & 12 \\ \hline 0 & 6 & 7 & 13 \\ \hline 2 & 10 & 7 & 6 \\ \hline \end{array}.$$

Exercise 1.5

1. Find the spatial resolution of an image if the scene of size $4m$ by $4m$ is represented by a 256×256 image.
2. Find the spatial resolution of an image if the scene of size $10km$ by $10km$ is represented by a 4096×4096 image.
3. Find the spatial resolution of an image if the scene of size $7mm$ by $7mm$ is represented by a 1024×1024 image.

Exercise 1.6

Determine the actual physical measurement in millimeters of an image with 1400 rectangular pixels and a resolution of 72 dpi.

Exercise 1.7

Propose an algorithm to map a rectangle to a disk (image warping).