Exercise 2.1

Given an 8×8 image whose grayscale values satisfy:

$$f(i, j) = |i - j|$$

Provide the output image when applying the following filters:

- 3×3 median filter ; 5×5 max filter 3×3 max filter
- Gaussian filter with standard deviation $\sigma = 0.5$ and size 3×3
- Gaussian filter with standard deviation $\sigma = 2$ and size 5×5

Use different strategies to handle the boundaries.

Exercise 2.2

Let I be an image of size 4×4, encoded with 4 bits/pixel:

$I =$

7	3	4	1
1	2	0	3
4	2	2	1
0	3	5	1

(i) Calculate the result of applying the following two transformations:

$$s = T_1(r) = 5\sqrt{r}, \quad s = T_2(r) = 15 - 2r$$

(ii) Calculate the following criteria: MAE, MSE, and $PSNR$.

Exercise 2.3

An image has a normalized histogram for which the following analytical form has been found:

$$h(r) = 6(r - r^2), \quad r \in [0, 1]$$

We assume here that r corresponds to a gray level (where 1 corresponds to white and 0 to black).

- Roughly sketch this histogram and specify the mean of the gray levels of this image both qualitatively and by calculation.
- Why could $\frac{1}{2}h(r)$ not be a normalized histogram curve?
- Determine the transformation $s = T(r)$ that would allow equalizing $h(r)$.
- Is information lost when performing histogram equalization on a digital image?

Exercise 2.4

Let X be a continuous random variable whose law admits the probability density f_X, and let T be a monotonic and differentiable function with $Y = T(X)$.

(i) Show that the density of Y is given by:

$$f_Y(y) = \left|\frac{1}{T'(T^{-1}(y))}\right| \cdot f_X(T^{-1}(y))$$

(ii) Determine the transformation T such that the variable Y follows any given distribution.

(iii) Apply histogram equalization to the following image (provide all necessary details):

$I =$

10	12	11	9	12
15	11	9	10	6
13	15	8	12	6
9	6	7	11	8
7	2	1	1	0

(iv) Plot the histograms of the original image and the equalized image.

(v) Compute the norm of the difference between the original and the equalized images.

Exercise 2.5

We are given an image I represented by the following table:

$I =$

175	150	114	86	79
156	119	91	80	113
132	93	80	96	174
96	85	87	165	193
87	82	153	192	194

(i) Apply a 3×3 max filter to I (the edge handling strategy must be explained).

(ii) Compute the Laplacian of the image I: ΔI.

(iii) Apply the following filter to the image I (specify the edge handling strategy):

0	-1	0
-1	5	-1
0	-1	0

(iv) What quantity is approximated by this filter?

Exercise 2.6

Apply histogram equalization to the following image (give all the necessary details).

$I =$

10	12	11	9	12
15	11	9	10	6
13	15	8	12	6
9	6	7	11	8
7	2	1	1	0

Exercise 2.7

Consider the image I represented by the following table:

$I =$

75	75	75	90	90
85	75	111	90	110
85	90	110	110	90
75	75	120	85	90
75	60	60	122	120

(i) Apply the median filter on the four neighbors and a Gaussian filter of size 3×3 with variance 0.8 to the image I. Use two different edge handling strategies. What are the theoretical and practical differences between the two?

(ii) Apply histogram equalization to I while maintaining the same dynamic range.

Exercise 2.8

Apply a histogram specification of the images:

Reference =

11	13	0	13
7	0	2	5
15	15	1	15
15	4	11	0

, Input =

13	14	2	14
10	2	5	9
15	15	3	15
15	8	13	1

Apply the following filters: Prewitt, Sobel, Roberts, and a contrast enhancement.

Exercise 2.9

What do the following convolution kernel filters do? (Provide a numerical example if necessary)

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad \frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \quad \frac{1}{16}\begin{bmatrix} -1 & -2 & -1 \\ -2 & 12 & -2 \\ -1 & -2 & -1 \end{bmatrix}$$

What is the condition on the coefficients for the filtering to be low-pass?