Exercise 3.1

Consider a discrete 4-periodic signal $x(n)$. Express x in terms of complex exponential and determine the DFT, $X(k)$. Compare the results.

$$x_1(n) = 1 + 3\cos\left(\frac{2\pi}{4}n + \frac{\pi}{3}\right) + 2\cos\left(2\frac{2\pi}{4}n\right),\quad x_2(n) = 2 + \cos\left(\frac{2\pi}{4}n + \frac{\pi}{6}\right) + \cos\left(2\frac{2\pi}{4}n\right)$$

Exercise 3.2

Using the DFT formula, compute the DFT of the following signals and verify the inversion and Parseval's formulas.

- $\{x(0) = 3, x(1) = \sqrt{3}, x(2) = 1, x(3) = -\sqrt{3}\}$
- $\{x(0) = 4, x(1) = 0, x(2) = 0, x(3) = 0\}$
- $\{x(0) = 2, x(1) = 2, x(2) = 2, x(3) = 2\}$
- A signal of size 8, obtained by sampling the function

$$f(n) = 2\cos\left(2\frac{2\pi}{8}n - \frac{\pi}{3}\right) = \cos\left(2\frac{2\pi}{8}n\right) + \sqrt{3}\sin\left(2\frac{2\pi}{8}n\right).$$

Find the same results using the DFT matrix.

Exercise 3.3

We consider a discrete 4-periodic image $x(m,n)$. Express x in terms of complex exponentials and derive the coefficients of its 2D DFT, $X(k,l)$. Verify the inversion formula and Parseval's theorem. Find the least squares error if x is represented by its DC component with the values $X(0,0)$, $0.9X(0,0)$, and $1.1X(0,0)$.

$$x(m,n) = 1 + 2\cos\left(\frac{2\pi}{4}(m+n) - \frac{\pi}{3}\right) + \cos\left(2\frac{2\pi}{4}(m+n)\right)$$

$$x(m,n) = 2 + 2\cos\left(\frac{2\pi}{4}(m+2n) + \frac{\pi}{3}\right) - \cos\left(2\frac{2\pi}{4}(m+n)\right)$$

Exercise 3.4

Find the 2D DFT of the images x and h using the row-column method. Reconstruct the input from the 2D DFT coefficients. Verify Parseval's theorem. Express the magnitude of the DFT in the centered format using the scale $\log_{10}(1 + |X(k,l)|)$.

$$x(m,n) = \begin{array}{|c|c|c|c|}\hline 112 & 148 & 72 & 153 \\ \hline 120 & 125 & 30 & 99 \\ \hline 95 & 120 & 89 & 33 \\ \hline 170 & 99 & 109 & 40 \\ \hline\end{array}, \quad h(m,n) = \begin{array}{|c|c|c|c|}\hline 164 & 127 & 117 & 59 \\ \hline 154 & 122 & 104 & 83 \\ \hline 129 & 136 & 100 & 360 \\ \hline 117 & 128 & 80 & 48 \\ \hline\end{array}$$

Find:
(a) The periodic convolution of x and h.
(b) The periodic correlation of x and h, and of h and x.
(c) The autocorrelation of x.

Exercise 3.5

Calculate the convolution of $x(n), n = 0, 1, \ldots$ and $h(n), n = 0, 1, \ldots$ using the DFT and IDFT. Verify the result by directly applying the convolution formula. Zero-padding is used.

(i) $x(n) = \{2, 1, 3\}$ et $h(n) = \{1, -2\}$, **(ii)** $x(n) = \{-1, 3\}$ et $h(n) = \{1, 3, 2\}$.

(iii) $x(n) = \{4, -1\}$ et $h(n) = \{-3, 1, -2\}$, **(iv)** $x(n) = \{-1, 2, 3\}$ et $h(n) = \{-2, 3\}$.

(v) $x = \begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 2 & -3 & 1 \\ \hline 1 & 2 & 1 \\ \hline\end{array}, \ , \ h = \begin{array}{|c|c|}\hline 2 & -1 \\ \hline 1 & 2 \\ \hline\end{array},$ **(vi)** $x = \begin{array}{|c|c|c|}\hline 2 & 2 & 3 \\ \hline 2 & -3 & -1 \\ \hline 1 & -2 & 1 \\ \hline\end{array}, \ , \ h = \begin{array}{|c|c|}\hline -2 & -1 \\ \hline 1 & 3 \\ \hline\end{array}$

Exercise 3.6

Using the DFT and IDFT, calculate the convolution of $x(m,n)$ with a 3×3 Gaussian low-pass filter with $\sigma = 1$. Assume periodic boundary conditions.

$$x_1(m,n) = \begin{array}{|c|c|c|c|}\hline 2 & 1 & 3 & 4 \\ \hline 1 & 1 & 4 & 2 \\ \hline 1 & -1 & 2 & -2 \\ \hline 3 & 2 & -2 & 1 \\ \hline\end{array}, \ , \quad x_2(m,n) = \begin{array}{|c|c|c|c|}\hline 1 & 2 & 3 & 4 \\ \hline 2 & 1 & 1 & 4 \\ \hline 1 & -1 & 0 & -2 \\ \hline 0 & 2 & -2 & 1 \\ \hline\end{array}$$

Exercise 3.7

We consider the image

$$I = \begin{array}{|c|c|c|c|}\hline 2 & 0 & 1 & 0 \\ \hline 1 & 1 & 0 & 1 \\ \hline 1 & 0 & 0 & 1 \\ \hline 2 & 1 & 2 & 3 \\ \hline\end{array}$$

(i) Calculate the 2D DFT of I.

(ii) State Parseval's theorem and verify it for the image I.

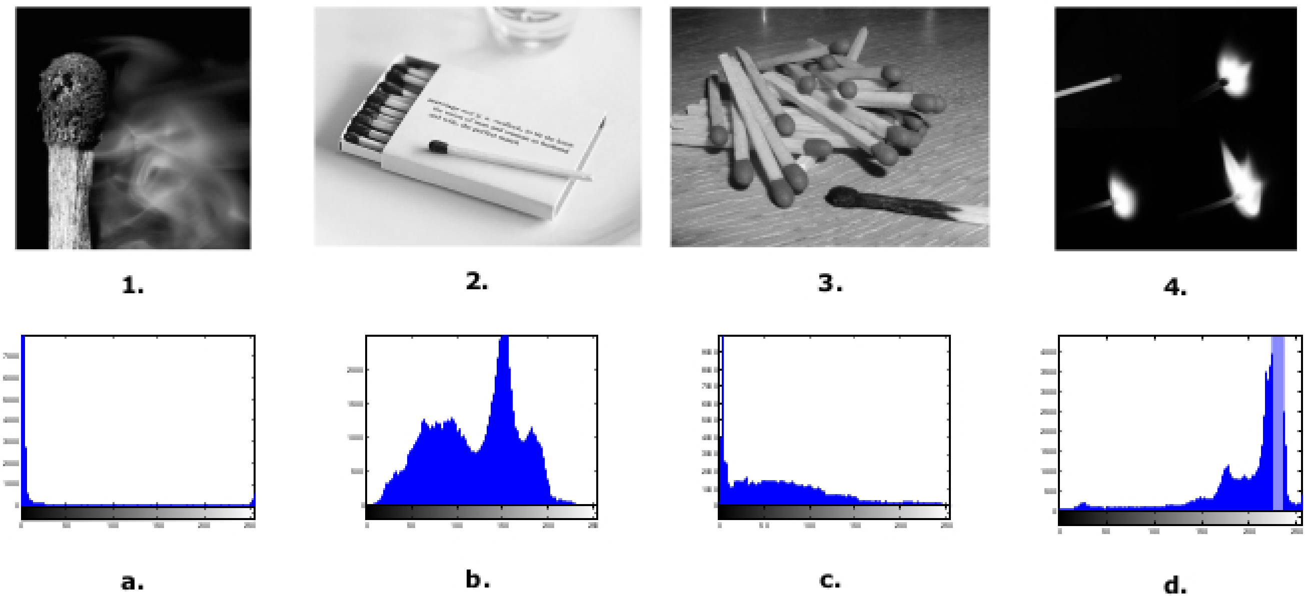
(iii) Apply the filter h to the image I in the frequency domain.

(iv) Apply the DCT2 and verify its inverse. Is Parseval's theorem still satisfied?

(v) Calculate the 1-level Haar Wavelet.

Exercise 3.8

Match which image 1, 2, 3, or 4 corresponds to the amplitude spectrum a, b, c, or d.



Exercise 3.9

Compute the DFT of the column vector $x(m) = \{0.2741, 0.4519, 0.2741\}$ and the row vector $x(n) = \{0.2741, 0.4519, 0.2741\}$. Using the separability theorem, verify that the product of the vectors in the time domain and the 2-D IDFT of the product of their individual DFTs are the same.

Exercise 3.10

Using the DFT and IDFT, find: (a) the periodic convolution of x(m, n) and h(m, n), (b) the periodic correlation of x(m, n) and h(m, n), and h(m, n) and x(m, n), (c) the autocorrelation of x(m, n).

$$x(m,n) = \begin{array}{|c|c|c|c|}\hline 2 & 1 & 3 & 3 \\ \hline 1 & 0 & 1 & 2 \\ \hline 4 & 1 & 0 & 1 \\ \hline 2 & 0 & 1 & 2 \\ \hline \end{array}, \quad , \quad h(m,n) = \begin{array}{|c|c|c|c|}\hline -2 & 1 & 3 & 2 \\ \hline 1 & 1 & -1 & -2 \\ \hline 4 & 0 & 0 & -1 \\ \hline 1 & 0 & 2 & 2 \\ \hline \end{array}$$