Exercise 6.1

(i) Compare and contrast histogram equalization and Contrast Limited Adaptive Histogram Equalization (CLAHE). In which scenarios would CLAHE be more effective?

(ii) A noisy image is given with Gaussian noise of mean 0 and variance σ^2. Propose a method to denoise the image and justify your choice mathematically.

Exercise 6.2

One of the most useful filters is Unsharp Masking, which sharpens a blurry or out-of-focus image. We will briefly explain a technique to achieve this effect.

– Let the transformation be $g(x, y) = f(x, y) - \gamma \Delta f(x, y)$, where γ is a gain parameter. Write a function `unsharp` (to be tested in a practical session) that implements this function, taking an image and a gain as parameters. The Laplacian will be computed by convolution with the following kernel:

$$\begin{matrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{matrix}$$

– There is a clever way to implement the Laplacian of Gaussian filter using two Gaussian functions. Show that, by taking $k \sim 1$ (in practice, 1.6 is appropriate):

$$\Delta g_\sigma(x, y) \sim g_{k\sigma}(x, y) - g_\sigma(x, y)$$

where g_σ denotes the centered Gaussian function.

Exercise 6.3

(i) Decompose the 2D filter with the given kernel as the product of two 1D convolution filters.

$$\frac{1}{b^2 + 4a(a+b)} \begin{array}{|c|c|c|} \hline a^2 & ab & a^2 \\ \hline ab & b^2 & ab \\ \hline a^2 & ab & a^2 \\ \hline \end{array}$$

(ii) Compute the complexity of the implementation with and without decomposition.

(iii) What is the condition on the coefficients for the filter to be low-pass?

Exercise 6.4

Consider an image I represented by the following matrix:

10	12	2	1
10	14	5	2
13	12	13	4
10	12	14	3

(i) Apply a mean filter and a Gaussian filter (3x3 with $\sigma = 2$) (zero-padding).

(ii) Apply a median filter (3x3) using mirror extension for handling borders. What is the role and benefit of a median filter, compared to other filters?

Exercise 6.5

Consider an image I and a structuring element SE (centered in the middle):

$$I = \begin{array}{|c|c|c|c|c|}\hline 175 & 150 & 114 & 86 & 79 \\ \hline 156 & 119 & 91 & 80 & 113 \\ \hline 13 & 93 & 80 & 96 & 174 \\ \hline 96 & 85 & 87 & 165 & 193 \\ \hline 87 & 82 & 153 & 192 & 194 \\ \hline \end{array}, \quad SE = \begin{array}{|c|c|c|}\hline 1 & 1 & 0 \\ \hline 0 & 1 & 0 \\ \hline 1 & 0 & 0 \\ \hline \end{array}$$

(i) Apply erosion followed by dilation. Partial masking should be applied to the border pixels.

(ii) Describe and explain the nature and purpose of the filter as defined.

Exercise 6.6

An image has been degraded by noise during transmission. This results in scratches, that is, white lines (value 255) with a thickness of 1 or 2 pixels. Propose a method to remove these defects: explain the principle of the method (justifying it) and provide its algorithm.

Exercise 6.7

We have an image I represented by the following matrix:

$$I = \begin{array}{|c|c|c|c|c|c|c|c|c|c|c|c|}\hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ \hline 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ \hline 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline \end{array}$$

(i) Calculate the morphological gradients obtained with the structuring element:

$$SE = \begin{array}{|c|c|c|}\hline 1 & 1 & 0 \\ \hline 0 & 1 & 1 \\ \hline 0 & 0 & 1 \\ \hline \end{array}.$$